

A Mach-Zehnder Interferometer for a Two-Photon Wave Packet

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We propose an experiment that permits observation of the de Broglie two-photon wave packet behavior for a pair of photons, using a Mach-Zehnder interferometer. It is based on the use of pulsed lasers to generate pairs of photons via spontaneous parametric down-conversion and the post-selection of events. It differs from previous realizations by the use of a third time-correlated photon to engineer the state of the photons. The same technique can give us which-path information via an “interaction-free” experiment and can be used in other experiments on the foundations of quantum mechanics related to wave-particle duality and to nonlocality.

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I. INTRODUCTION

Recently, Jacobson *et al.* [1] described theoretically an interferometer capable of measuring the de Broglie wavelength of an ensemble of photons, which is given by λ/N , where λ is the wavelength of each photon taken individually, and N is the average number of constituent photons. In their proposal, they suggest the use of high-Q cavities. More recently, Fonseca, Monken, and Pádua [2], performed a Young double-slit experiment to demonstrate, in a practical way, how to measure the de Broglie wavelength of a two-photon wave packet. This experiment has helped us to understand the role of entanglement in experiments in which the de Broglie two-photon wave packet behavior is observed through the frequency of the fourth order interference fringes. Here we wish to discuss the idea of an experiment which is equivalent to a Mach-Zehnder (M-Z) interferometer for a two-photon wave packet. The scheme produces a two-photon entangled state inside the interferometer. The behavior of the two-photon wave packet can be explained in terms of Einstein-Podolsky-Rosen (EPR) correlations [3]. A similar idea can be used to create a M-Z interferometer for three photons. In this case the three photons will be in a Greenberger-Horne-Zeilinger (GHZ) state [4].

Interference fringes oscillating with a frequency corresponding to a two-photon wave packet interference have been observed in previous experiments. The question of whether or not these doubled frequencies actually correspond to the interference of a wave packet containing two photons is more subtle. If one wants to observe two-photon wave packet interference (TPWI), two main difficulties must be overcome. The first one is to keep the two photons together, when they are crossing the interferometer. The second one is to separate the one-photon signal from the two-photon one in detection. In Ref. [2], the pair of photons was kept together by preparing the input state. In Ref. [5], Martienssen *et al.* have observed TPWI with a Michelson type interferometer. In their experiment, the events in which two photons travel together through the interferometer were selected from the other possibilities. This was performed by making one of the arms of the interferometer larger than the

other. Thus, whenever each photon followed a different path in the interferometer, no coincidence was registered. In both experiments, the two-photon detection was performed by photon coincidence. A similar approach was used by Burlakov *et al.* [6], where state preparation was employed in the observation of the TPWI.

Our proposal is based on the technique of generation of photon pairs via spontaneous parametric down-conversion (SPDC) using pulsed pump lasers [7]. The use of pulsed pumping allows us to have a temporal correlation between two initially independent photons, thereby engineering entangled states with more than two photons. Our approach can be used to investigate questions related to the foundations of Quantum Mechanics, such as wave-particle duality and nonlocality, for example.

II. UTILIZING THREE CORRELATED PHOTONS TO OBSERVE TWO-PHOTON WAVE PACKET INTERFERENCE

The experiment we are proposing is represented in Fig.1. Two photons with equal polarizations, which can be generated via type-I SPDC, follow path *a* and arrive together at beam-splitter (B.S.) H_1 . The M-Z interferometer is built with four 50-50 beam splitters (H_1, H_2, H_3 , and H_4). There are three possibilities for the paths followed by the photons: (1) one photon follows path *b* and the other follows path *c*, (2) both photons follow path *b*, and (3) both follow path *c*. To discard the first possibility, a third photon, with the same polarization as the other two, is introduced via path *d*. The three temporally correlated photons can be generated using pulsed lasers [7], so that whenever one or two photons follow path *b* they arrive at H_2 together with the photon following path *d*. From the different events which can take place, we are only interested in those in which only one photon is registered by detector D_A . (To be sure that only one photon has been registered by detector D_A , we use a triple coincidence circuit, as represented in Fig.1.) In this case, there are two possibilities: (1) No photon follows path *e*, which implies that two photons follow path *c*, or (2) two photons follow path *e*. If only one photon follows path

b , after H_2 we will have two photons following the same direction [8,9], that is, both will either impinge on detector D_A or follow path e , and no triple coincidence will be registered. To complete the interferometer for two photons, we introduce B. S. H_5 and use a triple coincidence circuit, so that only events in which two photons emerge together from the interferometer will be considered. In these circumstances, the gated two-photon detection rate R_{11} at site 1 is proportional to $1 - \cos 2\phi$. (The triple coincidence detection rate at sites 3,5 and 6 allows us to observe this dependence on 2ϕ , since it is proportional to R_{11} .) To see this, let us write the symmetrized three-photon initial state as [10]

$$N(|a\rangle|a\rangle|d\rangle + |a\rangle|d\rangle|a\rangle + |d\rangle|a\rangle|a\rangle), \quad (1)$$

where N stands for the normalization factor and $|a\rangle$ ($|d\rangle$) represents a photon coming via path a (d). The action of the B. S. s can be represented as $|a\rangle \rightarrow (1/\sqrt{2})(|b\rangle + i|c\rangle)$, and so on. It is then easy to show that the three-photon state, after H_2 and H_3 , is given by

$$N[|f\rangle|f\rangle + |e\rangle|e\rangle]|3\rangle + \text{symmetrization terms}], \quad (2)$$

where only the terms which in principle can lead to the triple coincidence detection we are interested in have been taken into account. (Please note that $|3\rangle$ represents the state of one photon following direction 3, it is *not* a three-photon Fock state.) The action of the phase shifter can be represented as $|e\rangle \rightarrow e^{i\phi}|e\rangle$. Hence, if a photon is detected at site 3, the state (2) is reduced to the two-photon state

$$N(|f\rangle|f\rangle + e^{2i\phi}|e\rangle|e\rangle). \quad (3)$$

From (3) we then obtain

$$N[(1 - e^{2i\phi})|1\rangle|1\rangle + i(1 + e^{2i\phi})|1\rangle|2\rangle + i(1 + e^{2i\phi})|2\rangle|1\rangle - (1 - e^{2i\phi})|2\rangle|2\rangle], \quad (4)$$

for the two-photon state after H_4 . From (4) we obtain the gated(or triple) coincidence detection rates:

$$R_{11} = R_{22} \propto P_{11} = P_{22} \propto |1 - e^{2i\phi}|^2 \propto 1 - \cos 2\phi \quad (5)$$

and

$$R_{12} = R_{21} \propto P_{12} = P_{21} \propto |1 + e^{2i\phi}|^2 \propto 1 + \cos 2\phi, \quad (6)$$

where the phase shift is given by $\phi = 2\pi\delta l/\lambda$, in which δl is a small difference between the two path lengths in the interferometer and λ is the wavelength of each photon. We see that in some respects the two photons behave as a single particle of wavelength $\lambda/2$. But when $\phi = 0$ and $R_{11} = 0$, this does not mean that the two photons are following direction 2. Actually, as we see from (5) and (6) each photon follows a different direction, that is, one photon goes to channel 1 and the other to channel 2.

III. FRINGES WITH DOUBLED FREQUENCY BUT WITHOUT TWO-PHOTON WAVE PACKET INTERFERENCE

Although in agreement with de Broglie's relation, $\lambda = h/p$, it may sound a bit surprising that two photons when moving together have a wavelength which is half the wavelength of each photon taken individually, since there is no interaction keeping the photons together. The physical property connecting the photons is entanglement. The pair of photons is not in an entangled state initially, at the interferometer entrance. The entanglement is induced by the third photon. In order to provide a clearer understanding of this aspect, let us consider a modified version of the experiment represented in Fig. 1, as represented in Fig. 2. Now we are interested in the situation in which one photon is detected at 3 and the other two impinge on H_4 and H_5 , respectively. From the previous discussion, we see that the third photon coming via path d acts as a sort of "catalyst", inducing entanglement between the other two photons (actually, a photon has no individuality, and it makes no sense to say that the photon at site 3 is the *same* photon which came via path d , so it is an abuse of language to refer to the "other two photons"). Since, in our experiment, the two photons always follow the same path in the first interferometer, if one photon impinges on H_4 via path g (i), the other can only impinge on H_5 following path h (j). That is, we have a direction-entangled two-photon state. As is well known [11–17], this leads to the phenomenon of two photon interference and to the violation of a Bell inequality, a signature of quantum nonlocality [18]. To see this, let us consider in Fig. 2 the situation in which a photon has been detected at 3 and the remaining two-photon system has already been reflected at H_2 and H_3 but has not yet reached the next B. S. From (2) and (3), we see that the two-photon state is given by

$$N(|f\rangle|f\rangle + |e\rangle|e\rangle). \quad (7)$$

It is then easy to see that after the phase shifters the two-photon state is

$$N[|i\rangle|j\rangle + |j\rangle|i\rangle + e^{i(\phi_1+\phi_2)}|g\rangle|h\rangle + e^{i(\phi_1+\phi_2)}|h\rangle|g\rangle], \quad (8)$$

where the terms in which the two photons follow the same path together are not included, since we are not interested in these events. Thus, after H_4 and H_5 the two-photon state will be

$$N\left\{ \left[1 - e^{i(\phi_1+\phi_2)} \right] (|1\rangle|5\rangle - |2\rangle|6\rangle) + i \left[1 + e^{i(\phi_1+\phi_2)} \right] (|1\rangle|6\rangle + |2\rangle|5\rangle) \right\}, \quad (9)$$

where the symmetrization terms have not been included, since they add no new information to our discussion.

Comparing the expressions (4) and (9) we see that the experiment represented in Fig. 1 is a degenerate case of the experiment represented in Fig. 2, which is similar to the experiments discussed in [11] and [17]. Therefore, the behavior of the two photons as a single entity can be understood as a consequence of the nonlocal correlations of the entangled photons. The interference fringes depend on $\phi_1 + \phi_2$. When $\phi_1 = \phi_2 = \phi$, they will oscillate with 2ϕ as in the TPWI. However, each photon impinges on a different B. S..

IV. FRINGES WITH DOUBLED FREQUENCY FOR SEPARATED PHOTONS

The previous discussion raises an interesting point, which shows how the photon picture can sometimes be misleading [19]. For example, let us consider the experiment represented in Fig. 1, but without the third photon coming via path d and with H_5 removed. We are only interested in the situation in which the two photons coming via path a arrive at H_4 . Therefore, H_2 and H_3 now represent perfect mirrors. This is an experiment in which Dirac's dictum "each photon interferes only with itself" [20] must apply. That is, if a light beam is sent to the interferometer via path a , the intensity observed at sites 1 and 2 will only depend on the phase ϕ , not on the mean number of photons in the beam. However, in our experiment we might be tempted to assume that we either have each photon following a different path or we have both photons following the same path together. In the first case, they would arrive together at H_4 and follow together the same direction [8,9], which could, with equal probability, be either 1 or 2. In the second, their behavior would depend on 2ϕ , as we have just seen. However, the interesting thing is that, although these two different behaviors can be discerned from the quantum mechanical formalism, the final result, given by the probabilities of detection, is the same as we would observe by considering the photons as totally independent. That is, we would observe the same result if we first sent a photon and detected it, and only then sent and detected the second photon. This can be seen as follows. After H_2 , H_3 , and ϕ , the initial two-photon state $|a\rangle|a\rangle$ becomes

$$\frac{1}{2} (-e^{2i\phi}|e\rangle|e\rangle - ie^{i\phi}|e\rangle|f\rangle - ie^{i\phi}|f\rangle|e\rangle + |f\rangle|f\rangle). \quad (10)$$

Hence, after H_4 we obtain

$$\begin{aligned} & \frac{1}{4} [(1 - 2e^{i\phi} + e^{2i\phi}) |2\rangle|2\rangle - i(1 - e^{2i\phi}) |2\rangle|1\rangle \\ & - i(1 - e^{2i\phi}) |1\rangle|2\rangle - (1 + 2e^{i\phi} + e^{2i\phi}) |1\rangle|1\rangle] \end{aligned} \quad (11)$$

for the two-photon state. From (11) we see that the probability amplitude for having the two photons being detected in the same channel depends on 2ϕ (which corresponds to both photons following the same path together)

and on ϕ (which corresponds to each photon following a different path). On the other hand, the probability amplitude for having each photon being detected in a different channel depends only on 2ϕ , since whenever they follow different paths they emerge from H_4 in the same channel. Nevertheless, the detection probabilities are exactly the same as we would have if instead of having the two photons arriving together at the interferometer we had them arriving at totally independent and different times. To see this, let us calculate the detection probabilities in an ideal situation. If only one photon is sent to the interferometer, the detection probability amplitudes are given by

$$\begin{aligned} A_1 &= \frac{1}{2}(1 + e^{i\phi}) \text{ and} \\ A_2 &= \frac{1}{2}(1 - e^{i\phi}). \end{aligned} \quad (12)$$

Thus, the probabilities are given by

$$\begin{aligned} P_1 &= |A_1|^2 = \frac{1}{4}(1 + e^{-i\phi})(1 + e^{i\phi}) \text{ and} \\ P_2 &= |A_2|^2 = \frac{1}{4}(1 - e^{-i\phi})(1 - e^{i\phi}). \end{aligned} \quad (13)$$

Considering the photons as independent, the probabilities of "coincident" detections (in this case, we have used the term "coincidence" for detections of individual photons at the same output port, P_{11} and P_{22} , or at different output ports, P_{12} and P_{21}) are given by

$$\begin{aligned} P_{11} &= P_1^2 = \frac{1}{16}(1 + e^{-i\phi})^2(1 + e^{i\phi})^2, \\ P_{12} &= P_{21} = P_1 P_2 = \frac{1}{16}(1 - e^{-2i\phi})(1 - e^{2i\phi}) \text{ and} \\ P_{22} &= P_2^2 = \frac{1}{16}(1 - e^{-i\phi})^2(1 - e^{i\phi})^2. \end{aligned} \quad (14)$$

On the other hand, using (11) we see that

$$\begin{aligned} A_{11} &= \frac{1}{4}(1 + e^{i\phi})^2, \\ A_{12} &= A_{21} = \frac{1}{4}(1 + e^{i\phi})(1 - e^{i\phi}) \text{ and} \\ A_{22} &= \frac{1}{4}(1 - e^{i\phi})^2. \end{aligned} \quad (15)$$

Hence,

$$\begin{aligned} P_{11} &= |A_{11}|^2 = P_1^2 \\ P_{12} &= P_{21} = |A_{12}|^2 = P_1 P_2 \text{ and} \\ P_{22} &= |A_{22}|^2 = P_2^2. \end{aligned} \quad (16)$$

We obtain the same result.

From (11), we see that interference patterns with oscillations depending on 2ϕ can be observed by performing coincidence measurements between output channels 1 and 2. Because the photons split at the output of the

interferometer, it would be questionable to interpret this result as a TPWI. Moreover, from (16), we see that we can also have oscillating patterns depending on 2ϕ if we send one photon at a time through the interferometer. This would be even harder to interpret as a TPWI.

For the three-photon version of the M-Z interferometer we have just presented, the interference patterns depending on 2ϕ are obtained, and the interpretation in terms of TPWI is always possible.

V. THREE-PHOTON WAVE PACKET INTERFERENCE

The same idea can be extended to build an interferometer for a three-photon wave packet. In this case, three photons impinge on H_1 , a fourth photon impinges on H_2 , a fifth photon impinges on H_3 , and we register the fivefold coincidence detection at sites 3, 4, 5, 6, and 7 (Fig. 3). It is easy to see that now we have three photons following the same path together. If only one photon is detected at site 3, it is possible to infer that either no photon, two photons, or three photons follow path e . Similarly, if only one photon is detected at site 4, either no photon, two photons, or three photons follow path f . Since three photons impinge on H_4 , the only possibilities are no photon following path e and three photons following path f , or three photons following path e and no photon following path f . Moreover, the symmetry of the situation allows us to conclude that the probabilities for these two possibilities are the same. Now we have a 3ϕ dependence in the interference. It is easy to see that the two photons coming via paths d and h induce entanglement between the three photons impinging on H_4 , generating a GHZ state.

VI. DISCUSSION

In this paper we have proposed a way to achieve, at least for the cases of two and three photons, what Yamamoto has imagined, that is, a B.S. that splits a many-photon wave packet as a whole. We have also shown that the many-photon wave packet behavior is a consequence of entanglement. From this point of view, we see that there is a much simpler way to entangle the photons in the interferometer: instead of having the two photons coming via path a , we can have one photon coming via path a and the other coming perpendicularly to path a . After H_1 they will necessarily follow the same direction. However, this procedure cannot be extended to the situation in which we have a three-photon wave packet, and it is not a realization of Yamamoto's idea. We have also seen that the assumption according to which the events are "out there" and we are selecting only those in which the two photons go together may be misleading. Naturally, our explanation of the many-photon wave packet

behavior in terms of EPR correlations can in principle be extended to any system of particles, including atoms and molecules, for example, since to the extent that the particles have to go together, either along one path or the other, after a B. S., they are in a direction-entangled state.

We would like to point out that the introduction of an additional photon via path d , as we are proposing (Fig. 1), can give us "interaction-free" which-path information. For example, if we have only one photon coming via path a and another coming via path d , whenever only one photon is detected at site 3 it is possible to infer that the photon coming via path a has followed path c . In this way, it is possible to obtain information about the path followed by the first photon without directly interacting with it. This suggests interesting new experiments on the foundations of quantum mechanics, more specifically, on wave-particle duality and pilot wave interpretation, as well as on quantum nonlocality, to be discussed in a paper to follow.

VII. CONCLUSION

A new version of a Mach-Zehnder interferometer is presented. Two temporally correlated photons are sent through the input port, while a third one is sent through another port. As a result, interference patterns oscillating with a frequency two times larger than that of a single photon are observed when triple coincidence is detected. We show that they can be interpreted as two-photon wave packet interference. The same kind of pattern can be observed for different experimental arrangements, but in most of them the interpretation in terms of two-photon wave packet interference is not possible. Another version of the M-Z interferometer for three-photon wave packet interference is also presented. The schemes can be useful for studying wave-particle duality and quantum nonlocality.

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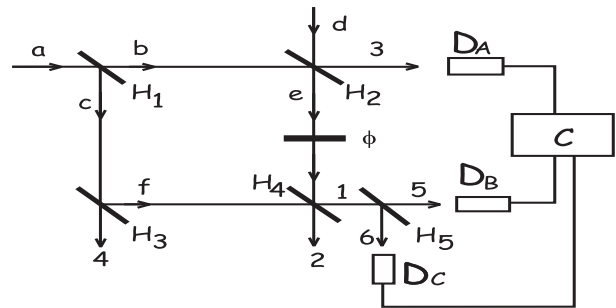


FIG. 1. Sketch of a M-Z interferometer for observing Two-Photon Wave Packet Interference.

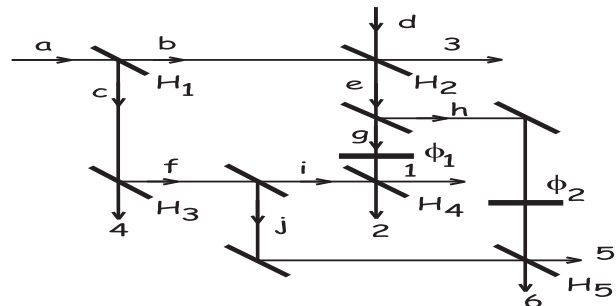


FIG. 2. An extension of the experiment represented in Fig. 1, to disclose its nonlocal features.

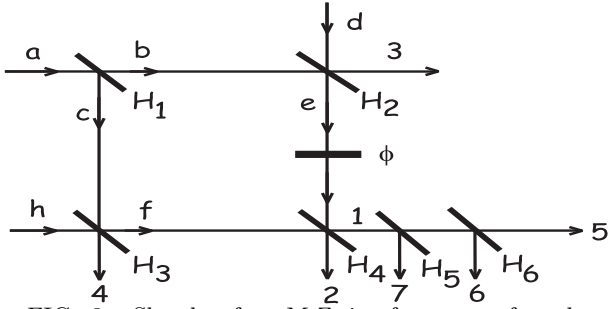


FIG. 3. Sketch of a M-Z inteferometer for observing Three-Photon Wave Packet Inteference.